This paper was presented at:

The 141st SMPTE Technical Conference and Exhibition
Displays for the Theater and Home Session
November 20, 1999

Cinema Projection Distortion

Ronald A. Petrozzo
Stuart W. Singer
Schneider Optics, Inc,
Hauppauge, New York

Abstract

This paper addresses projected image Distortion in a cinema setting. Several types of distortions are defined and examples are presented to show that the degree of distortion in the image can be calculated based on the theater geometry as well as the film format and projector lens specifications. This paper concentrates on inherent distortion values that typically affect the overall shape of the picture area and the relative shapes of all subjects being viewed as well as the audience's perception of imaged picture quality. Since distortion changes only the shape of objects in the picture and not how well the viewer can see them, there is little basis for objectively derived design limits. Further, when viewing a *real scene* that does not contain a perfect rectilinear test grid, the eye accommodates small amounts of distortion, which is therefore not perceived. However, moderate to severe amounts of distortion, and particularly distortion in *memory objects* such as curved horizons and leaning or curved flagpoles, is disturbing to viewers.

INTRODUCTION

This paper concentrates on inherent Distortion Values that typically effect the overall shape of the picture area and the relative shapes of all subjects being viewed as well as the audience's perception of imaged picture quality.

The term **Distortion** can have several different meanings, depending on the context of its use. For this paper, Distortion is limited to the following five forms:

- **Keystone Distortion**: Horizontal and/or Vertical Trapezoidal picture shape.
- **Anamorphic Stretch**: Horizontal and/or Vertical Image Stretch dependent on the keystone geometry.
- **Geometric Distortion**: Horizontal distortion dependent on screen curvature.

In all cases, distortion is defined as "the change in shape of the projected image relative to the shape of the projected image on a flat screen with the projector being perpendicular to the screen and centered." Any distortions based on viewing angle or viewer position in the theater are not addressed in this paper.

Since distortion changes only the shape of objects in the picture and not how well the viewer can see them, there is little basis for objectively derived design limits. In a theater setting, the eye will normally accommodate small amounts of distortion, which is therefore not perceived. However, when moderate to severe amounts of distortion are present, particularly when images of straight line objects such as flagpoles and the horizon are visible, the effect becomes apparent to the viewer and distracts from the overall presentation of the viewed image. Based on our collective experience, research and testing, we have found that when each of the above listed distortions have a value less then 5% (absolute value) the audience does not perceive any picture degradation due to these effects and will find the picture to be pleasing.

KEYSTONE DISTORTION & ANAMORPHIC STRETCH

Keystone Distortion and Anamorphic Stretch are forms of geometric distortion that bring about a trapezoidal image of a nominally rectangular picture. They are usually produced when a picture is projected from a position such that the line of sight or optical axis of the projector is not precisely orthogonal to the screen. In this situation, Keystone Distortion and Anamorphic Stretch occur because the film plane and image plane are no longer parallel to each other. **Figure 1**, below, shows an illustration of an extreme example of vertical keystone distortion.

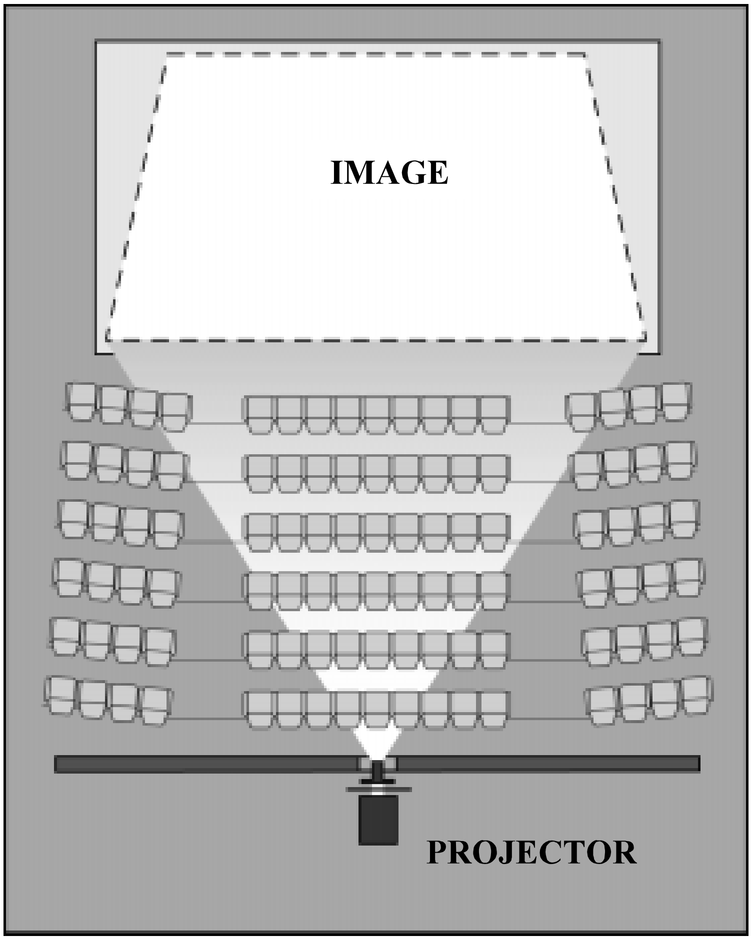


Figure 1: Vertical Keystone Distortion

Figure 2, below, shows an illustration of an extreme example of simultaneous horizontal and vertical keystone distortion.

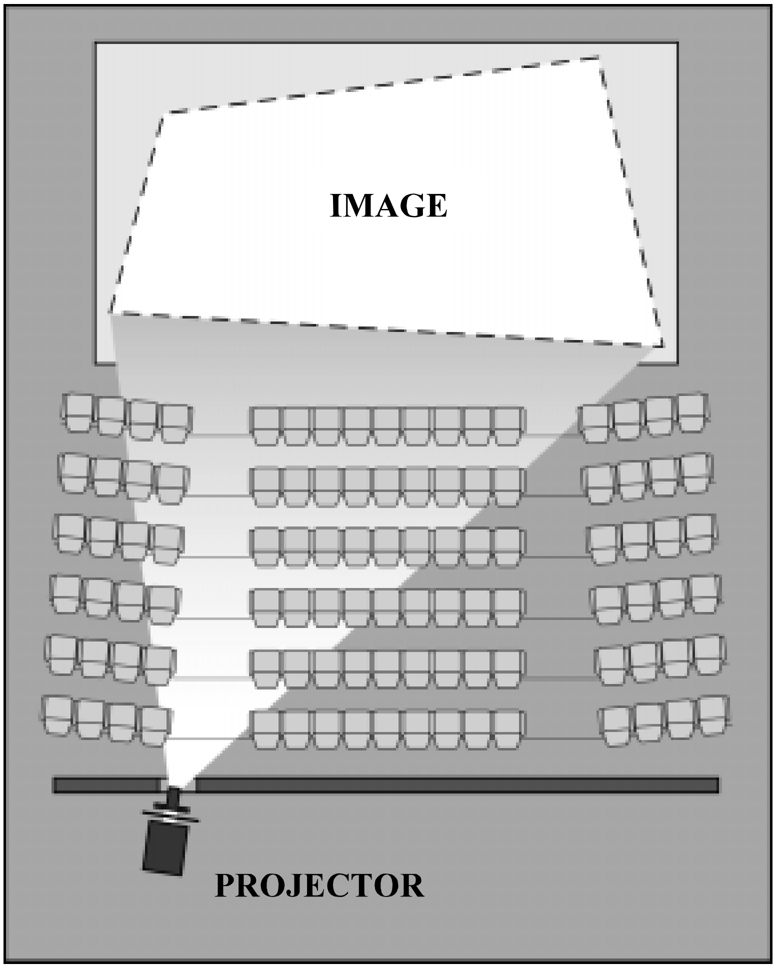


Figure 2: Simultaneous Horizontal and Vertical Keystone Distortion

The most typical example of these distortions is found when a projector is placed high up in a theater and tilted downward toward the screen (Refer to **Figure 3** below). The vanishing point, **V**, is defined such that a line through the projection lens parallel to the film would intersect the plane of the screen at a point high up above the roof of the theater. The undistorted image is represented by **AA**′ and the screen image by **BB**′. The screen image will be keystoned, and anamorphically stretched because **BB**′ is longer then **AA**′ in the vertical direction. It should be noted that this also applies in the horizontal direction if the projector is placed left or right of the screen centerline.

In short, keystone distortion and anamorphic stretch each have both a vertical and horizontal component.

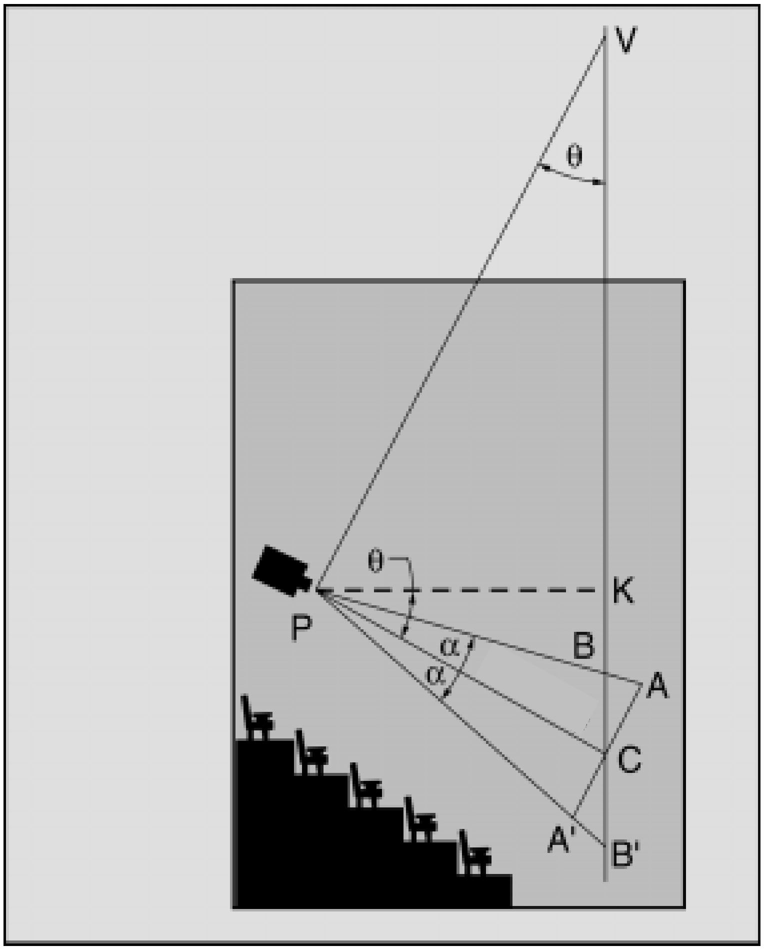


Figure 3: Projector / Screen Geometry

As an example, the following is a calculation of Anamorphic Stretch in the vertical direction. All values in the following refer to defined reference lengths and angles illustrated in **Figure 3**. For example **KB** refers to the distance defined from the point marked **K** to the point marked **B**.

Given:

Throw **= PK** = 100 feet = 1200"
Downward tilt of the projector = θ = 15°
Lens focal length = **f**′= 30mm = 1.181"
Film Format = 35mm 1.85 (0.825" x 0.446")

Note: In the following calculations all distance results are rounded to the nearest inch.

A) The 1/2 projected angle, α, in the vertical direction is calculated by the following formula

$$\alpha = \mathbf{Tan^{-1} [\tfrac{1}{2} Y' \div f']}$$

where **Y′** = film format size (please note the film orientation)

thus

α **= Tan-1 [½ ∗ 0.446 ÷ 1.181] = 10.69°**

B) Then we see that

KB = PK ∗ Tan (θ - α)

KB = 100' ∗ Tan (15° - 10.69°) = 7' 6"

and

KB′ = PK ∗ Tan (θ + α)

KB′ = 100' ∗ Tan (15° + 10.69°) = 48' 1"

KB′ is the projector height above the bottom of the image on the screen.

C) The vertical height of the screen image is

BB′= (KB′ - KB)

BB′= (48' 1"- 7' 6") = 40' 7"

D) **AA′**, the height of the undistorted image is given by

AA′ = 2 ∗ PC ∗ Tan (α)

where

PC = PK ∗ Sec (θ)

so

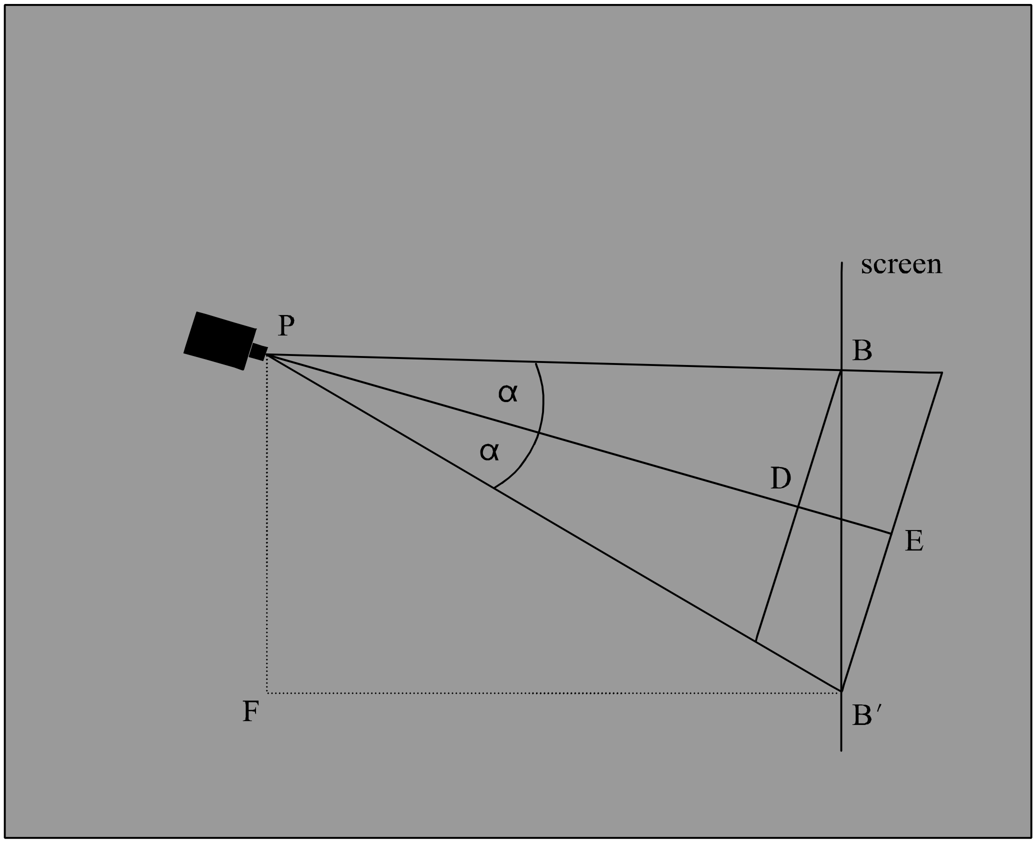
AA′ = 2 ∗ PK ∗ Sec (θ) ∗ Tan (α)

AA′ = 2 ∗ 100' ∗ Sec (15°) ∗ Tan (10.69°) = 39' 1"

E) The Anamorphic Distortion (Stretch) in the vertical direction is defined by

% Anamorphic Distortion = [(BB′ ÷ AA′) - 1] ∗ 100

% Anamorphic Distortion = [(40’ 7" ÷ 39’ 1") - 1] ∗ 100 = 3.8%



Note: The above calculations can also be applied to the horizontal direction, if your projector is displaced horizontally at an acute angle relative to the screen.

Figure 4: Projector / Screen Geometry for Keystone Calculation

As an example, the following is a calculation of the Keystone Distortion in the vertical direction. All dimensions refer to the geometry defined in **Figure 3** and **Figure 4**, above**. Figure 4** uses the same geometry as **Figure 3**.

PF (in **Figure 4**) is defined as the projector height relative to the bottom of the image. Comparing **Figure 3** and **Figure 4**, it can be seen that this value is given by the following:

PF = KB′ = 48’ 1"

Calculate **PD**, the distance from the projector to the center point of the plane perpendicular to the projector line of sight that intersects the screen at the top of the image:

Assume the following values are the same as the previous example:

α = **10.69°** = the 1/2 projected angle in the vertical direction

FB′ = PK = 100' = the Throw

Thus

PD = Cos (α) $*$ ((FB′)2 + (PF - BB′)2)1/2

PD = Cos (10.69°) $*$ ((100')2 + (48' 1" - 40' 7")2)1/2 = 98' 6"

Calculate **PE**, the distance from the projector to the center point of the plane perpendicular to the projector line of sight that intersects the screen at the bottom of the image:

PE = Cos (α) $*$ ((FB′)2 + PF 2)1/2

PE = Cos (10.69°) $*$ ((100')2 + (48' 1")2)1/2 = 109'

D) The 1/2 projected angle, γ, in the horizontal direction is calculated by the following formula

γ = Tan-1 [½ Y′ ÷ f′]

where **Y′** = film format size in the horizontal direction

thus

γ = Tan-1 [½ $*$ 0.825 ÷ 1.181] = 19.25°

E) Calculate the picture top width.

Top Width = 2 $*$ PD $*$ Tan (γ)

Top Width = 2 $*$ (98' 6") $*$ Tan (19.25°) = 68' 9"

F) Calculate the picture bottom width:

Bottom Width = 2 $*$ PE $*$ Tan (γ)

Bottom Width = 2 $*$ (109') $*$ Tan (19.25°) = 76′ 1"

G) The Keystone Distortion in the Vertical direction is defined as:

% Keystone = [1 - (Top Width ÷ Bottom Width)] $*$ 100

% Keystone = [1 - (68' 9" ÷76′ 1")] ∗ 100 = 9.6%

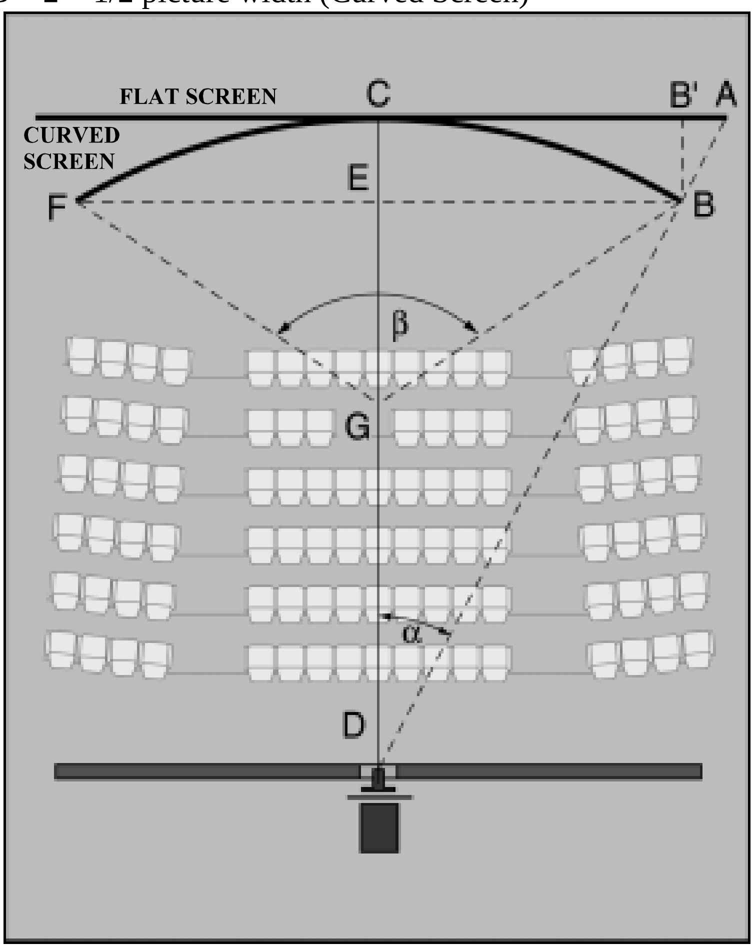
Note: The above calculations can also be applied to the horizontal direction, if your projector is displaced horizontally at an acute angle relative to the screen.

CURVED SCREEN DISTORTION

Today many theater screens are curved in an attempt to produce more even illumination and to add apparent depth to the picture. The overall goal is to make the picture more pleasant to the viewer. However, when a screen is curved, the actual viewed picture contains a form of geometric distortion. In all cases, the screen is curved in one orientation (horizontal) and concave to the viewers (with screen edges closest to viewers). Since the shape of the curve is a simple segment of a cylinder, the distortion that it yields is only in one orientation (horizontal).

As shown in **Figure 5**:

CA = 1/2 picture width (Flat Screen) No distortion
CB′ = FB ÷ 2 = 1/2 picture width (Curved Screen)



CD = Projection Distance from projector to screen center

Figure 5: Projector / Curved Screen Geometry

In the above diagram, **G** = location of the center of curvature of the screen where

CG = FG = BG = Curve

The distortion due to the curved screen is given by:

% Curved Screen Distortion = [(CB′ - CA) ÷CA] ∗ 100

The % Curved Screen Distortion will always be a negative value. This is due to the sign condition of the distortion where a negative value denotes “barrel” type distortion of the viewed picture.

For example:

Given:

Throw = 100**’**
Lens = 30mm = 1.181"
Film Format = 35mm 1.85 (0.825" x 0.446")
Curve = 2/3 Throw = 67 feet
Projector Down Angle = θ = 10 degrees

Calculate **CD** = Projection Distance, the straight line distance from the projector to the screen center

CD = (Throw) ∗ Sec (θ)

CD = 100’ ∗ Sec (10°) **= 101' 6"**

Note: The value **CD** = Throw, if there is no projector down angle.

B) Calculate the 1/2 projection angle in the horizontal direction:

The 1/2 projected angle (α) is calculated by the following formula

α = Tan-1 [½ ∗ **Y′ ÷ f′]**

where Y′ = film format size (please note the film orientation), thus

α = Tan-1 [½ ∗ **0.825 ÷ 1.181] = 19.25°**

C) Calculate **CA** = 1/2 picture width with respect to distance **CD** and flat screen

CA = CD ∗ Tan (α**)**

$CA = 101' 7" * \tan(19.25°) = 35' 5"$

D) Calculate total involved angle β

$\beta = 2 * ([\sin^{-1}((\sin(\alpha)) * (CD - Curve) \div Curve)] + \alpha)$

$\beta = 2 * ([\sin^{-1}((\sin(19.25°)) * (101' 6" - 67') \div 67')] + 19.25°) = 58.04°$

E) Calculate **FB**, the new image width at the center of the image which includes the curvature of the screen

$FB = [2 * Curve^2 * (1 - \cos(\beta))]^{1/2}$

$FB = [2 * (67')^2 * (1 - \cos(58.04°))]^{1/2} = 65'$

and

$CB' = EB = FB \div 2$

$CB' = 65' \div 2 = 32' 6"$

F) Calculate **CE**, the cord depth

$CE = Curve - [Curve^2 - (CB')^2]^{1/2}$

$CE = 67' - [(67')^2 - (32' 6")^2]^{1/2} = 8' 5"$

G) Calculate percent distortion

$\% \text{ Curved Screen Distortion} = [(CB' - CA) \div CA] * 100$

$\% \text{ Curved Screen Distortion} = [(32' 6" - 35' 6") \div 35' 6"] * 100 = -8.3\%$

Note: The number indicating the percentage is a negative value. This indicates Barrel shaped distortion at the screen.

DISTORTION TOLERANCE

Based on our collective experience, research and testing, we have found that when each of the above listed distortions have a value less then 5% (absolute value), the audience does not perceive any picture degradation due to these effects and will find the picture to be pleasing. This applies to real life imagery and not necessarily to a rectilinear test grid image. When viewing a motion picture in a theater, the viewed image is not simultaneously referenced with a "non-distorted" grid, so the observer cannot perceive the suggested tolerance values.

CONCLUSION

In this paper we have shown how to calculate five forms of distortion: vertical and horizontal Keystone Distortion, vertical and horizontal Anamorphic Stretch and Geometric Distortion due to screen curvature. Several examples were given to illustrate the geometry involved and the order of magnitude of the results that will be seen based on typical geometries. Design limits of 5% maximum for each of the types of distortions defined are proposed based on experience, research and previous testing. No distortion effects based on viewing angle were considered since these are dependent on the location of the observer in the theater. The distortions defined herein are only dependent on the physical geometry of the screen in reference to the projector position and orientation as well as the lens focal length.

About the authors:

Stuart W. Singer is a Senior Optical Engineer and Lens Designer with more than 20 years experience. He is Director of Technology for Schneider Optics, Inc. Mr. Singer holds a Bachelor of Science Degree in Physics from Hofstra University, Hempstead, NY.

Ronald A. Petrozzo is an Optical Engineer and Optical Systems Designer with more than 15 years experience. Mr. Petrozzo holds a Bachelor of Science Degree in Optics and a Master of Science Degree in Optics, both from the University of Rochester, Rochester, NY, as well as a Master of Science Degree in Computer Science from Polytechnic University, Farmingdale, NY.

REFERENCES

1) Smith, Warren J., Modern Optical Engineering, 2nd ed., 1990, McGraw-Hill, Inc.

2) Kingslake, Rudolf, Optical System Design, 1983, Academic Press, Inc.

3) Farrell, Richard J., & Booth, John M., Design Handbook For Imagery Interpretation Equipment, 1984, Boeing Aerospace Co.

4) Military Standardization Handbook, Optical Design, MIL- HDBK - 141, US Department of Defense.

5) Sasian, Jose M., “Image Plane Tilt in Optical Systems”, Optical Engineering / March 1992 / Vol. 31 No. 3, AT&T Bell Laboratories.

6) Venning, Albert L, Jr., “Trapezoidal Distortion in Projected Images”, Journal of the SMPTE / June 1968 / Vol. 77.

7) SMPTE Engineering Guideline EG 18-1994, Design of Effective Cine Theaters, March 1994.